LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – NOVEMBER 2007

BB 20

ST 5400 - APPLIED STOCHASTIC PROCESSES

Date : 03/11/2007 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

SECTION A

ANSWER ALL QUESTIONS. *(10 X 2 =20)*

01. Give an example for a discrete time and continuous state space stochastic process
02. Define : Covariance Stationary.
03. Give an example of an irreducible Markov chain.
04. Give an example for a stochastic process having "Stationary Independent Increments" ?
05. Mention the usefulness of classifying a set of states as closed or not in a Markov chain.
06. Define the term: Mean recurrence time.
07. When do you say a given state is "positive recurrent"?
08. Give an example for symmetric random walk...
09. Under what condition pure birth process reduces to Poisson process?
10. For what values of p and q the following transition probability matrix becomes a doubly stochastic matrix?

$$\begin{bmatrix} 1/4 & 3/4 \\ p & q \end{bmatrix}$$

SECTION B

Answer any FIVE questions *(5 X 8 =40)*

11. Consider the process $X(t) = A_0 + A_1 t + A_2 t^2$ where A_0, A_1 and A_2 are uncorrelated random variables with mean 0 and variance 1. Find mean and variance functions and examine whether the process is covariance stationary

12. Show that every stochastic process with independent increments is a Markov process

13. Show that a Markov chain is completely determined if its transition probability matrix and the distribution of X_o is known.

14. A player chooses a number from the set of all non negative integers. He is paid an amount equivalent to the number he gets. Write the TPM corresponding to his earnings, given that probability of getting the number is $\frac{1}{2^{i+1}}, i = 0,1,2,.....$

15. Consider the following Transition Probability Matrix explaining seasonal changes on successive days (S- Sunny, C-Cloudy)

Today

$$\begin{array}{lc} & \begin{array}{cccc} (S,S) & (S,C) & (C,S) & (C,C)r \end{array} \\ \text{Yesterday} \begin{array}{c} (S,S) \\ (S,C) \\ (C,S) \\ (C,C) \end{array} & \begin{bmatrix} 0.8 & 0.8 & 0 & 0 \\ 0 & 0 & 0,4 & 0.6 \\ 0.6 & 0.4 & 0 & 0 \\ 0 & 0 & 0.1 & 0.9 \end{bmatrix} \end{array}$$

Compute the stationary probabilities and interpret your results

16. A radioactive source emits particles at a rate of 5 per minute in accordance with a poisson process. Each particle emitted has a probability of 0.6 of being recorded, Find in a 4 minute interval the probability that the number of particles recorded is 10..

17. Given the transition probability matrix corresponding to the Markov chain with states {1,2,3,4}Find the probability distribution of n which stands for the number of steps needed to reach state 2 starting from the same and also find its mean. Offer your comments regarding the state 2

$$\begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

18. Obtain the differential equation corresponding to Poisson process

SECTION C

Answer TWO questions. *(2 X 20 =40)*

19. (a) Let $\{Z_i, i=1,2,....\}$ be a sequence of random variables with mean 0 and

$X_n = \sum_{i=1}^{n} Z_i$ is a Martingale. (10)

(b) Illustrate with an example basic limit theorem. (8)

20. (a) Show that for a process with independent increments $E[X(t)] = m_0 + m_1 t$ is linear in t (12)

(b) State and prove the reproductive property of Poisson process (8)

21. Explain the postulates of Yule-Furry process and find an expression for $P_n(t)$

22. Write short notes on the following

(a) Stationary Distribution (5)

(b) Communicative sets and their equivalence property (5)

(c) Interarrival time in Poisson process (5)

(d) Periodic States (5)

****